Double Angle Formulae

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$$

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

These formulae are very useful for expressing double an angle in terms of the original angle. The ≡ symbol means "identical to" (e.g. sine two alpha is identical to 2 sine alpha cos alpha). This symbols means the relationship is always true, regardless of the value of α.

For example, if we wanted to find $\sin 120$ we can use our double angle formulae.
We know that $2 \times 60 = 120$

$$\sin(2 \times 60) = 2 \times \sin(60) \times \cos(60)$$

We should know that $\sin(60) = \frac{\sqrt{3}}{2}$ and $\cos(60) = \frac{1}{2}$

$$\sin(120) = 2 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$$

Proof

We can use the addition formulae to prove these equations

Equation one

$$\sin(\alpha + \beta) \equiv \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

If we let $\alpha = \beta$

$$\sin(\alpha + \alpha) \equiv \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$$

You will notice that we are adding two of the same thing

$$\sin(2\alpha) \equiv 2\sin\alpha\cos\alpha$$

Equation two

$$\cos(\alpha + \beta) \equiv \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

If we let $\alpha = \beta$

$$\cos(\alpha + \alpha) \equiv \cos\alpha\cos\alpha - \sin\alpha\sin\alpha$$

This simplifies to

$$\cos(2\alpha) \equiv \cos^2\alpha - \sin^2\alpha$$

We can use the identity $\sin^2\alpha + \cos^2\alpha = 1$

$$\cos(2\alpha) \equiv (1 - \sin^2\alpha) - \sin^2\alpha$$

Which simplifies to

$$\cos(2\alpha) \equiv 1 - 2\sin^2\alpha$$

Or

$$\cos(2\alpha) \equiv \cos^2\alpha - (1 - \cos^2\alpha)$$

Which simplifies to

$$\cos(2\alpha) \equiv 2\cos^2\alpha - 1$$

All three of the highlighted equations above are acceptable, and different versions may be useful for different problems.

Equation three

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Let $\alpha = \beta$

$$\tan(\alpha + \alpha) = \frac{\tan\alpha + \tan\alpha}{1 - \tan\alpha\tan\alpha}$$

This simplifies to

$$\tan(2\alpha) = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

See also
- Addition Formulae

References
Attwood, G. et al. (2017). *Edexcel A level Mathematics - Pure - Year 2*. London: Pearson Education. pp.174-175